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An Improved Empirical Mode Decomposition Based On Particle Swarm Optimization

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Abstract: End effect is the main factor affecting the application of Empirical Mode Decomposition (EMD). This paper presents an improve EMD for decomposing short signal. First, analyzing the frequency components of signal to be decomposed, and construct the parameter equation with the amplitude and initial phase of signal as unknowns. Second, employing particle swarm optimization (PSO) to estimate the unknown parameters, and extending the inspected signal according to the obtained parameters. Thirdly, using EMD to decompose the extended signal into a series of intrinsic mode functions (IMFs) and a residual. The IMFs of original signal are extracted from these obtained IMFs. The correlation coefficients between the IMFs and the signal are calculated to judge the pseudo-IMFs. The simulation result shows that the presented method is effective and extends the application of EMD.

Key words: empirical mode decomposition, end effect, particle swarm optimization, short signal

I. Introduction

Empirical mode decomposition (EMD) is a signal process method first proposed by Huang [1] in 1998. EMD can decompose a given signal into a collection of Intrinsic Mode Functions (IMF) and a residue. The residue denotes the slowest frequency of signal, usually a trend of given signal. Every IMF denotes one frequency component of signal, usually local characteristics of given signal. EMD is an effective method for analyzing nonlinear and non-stationary signal. EMD method is widely used in image processing [2], fault diagnosis [3], earthquake signal processing [4], and so on. EMD method itself has a limitation called end effect. End effect directly influences application performance of EMD. Mirror image method, global statistics method, neural network method and auto regressive moving average (ARMA) model method have been proposed to control end effect [5]. Good performances have been achieved in some researches by using theses methods. In practice, we often need to analyze short signal containing few extreme points. If we use EMD to decompose this short signal directly, the decomposition result will be poor and the IMFs obtained can not accurately reflect the characteristics of decomposed signal. The improved EMD methods mentioned above also cannot work well.

In this paper, endpoint extension based Particle Swarm Optimization (PSO) is presented to improve EMD performance. In the following sections, the concept and steps of EMD are introduced; the causes of producing end effect and pseudo-IMF are analyzed; the concept and algorithm of PSO is introduced; improved EMD based PSO is proposed; the simulation is conducted to evaluate the performance of the proposed method.

II. Empirical mode decomposition

EMD can decompose a given signal into a collection of IMFs and a residue. An IMF is a function that satisfies two conditions [1]: (a) in the whole data set, the number of extrema and the number of zero crossing must either equal or differ at most by one; (b) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. Given a signal $x(t)$, the effective algorithm of EMD can be summarized as follows[1]:

1) Identify all extrema of $x(t)$;

2) Interpolate between maxima (respectively minima), ending up with some envelope $e_{max}(t)$ (respectively $e_{min}(t)$);

3) Compute the average $m(t)=(e_{max}(t) + e_{min}(t))/2$;

4) Extract the detail $d(t)=x(t)-m(t)$;

5) Iterate on the residual $r(t)$.

In practice, the above procedure has to be refined by a sifting process which amounts to first iterating steps 1) - 4) upon the detail signal $d(t)$, until the latter can be considered as zero-mean according to the standard deviation[1]

$$SD = \sum_{t=0}^{T}\left[\frac{|d_{k-1}(t) - d_k(t)|^2}{d_{k-1}^2(t)}\right], \tag{1}$$

where d_k(t) denotes the k-th detail during the sifting process. The typical value for SD can be set from 0.2 to 0.3. Once SD reaches the typical value, the detail d_k(t) can be considered as the effective IMF, the corresponding residual is computed by

$$r(t) = x(t) - d_k(t), \tag{2}$$

and step 5) applies. When the residual $r(t)$ becomes a monotonic function from which no more IMF can be extracted, we have

$$x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t), \tag{3}$$

where $c_i(t)$ (i=1,2,…,n) denotes IMF and $r_n(t)$ denotes the residue, and c_1(t) has the locally highest frequency in $x(t)$, while $r_n(t)$ contains the slowest frequency, usually a trend of $x(t)$.

III. End effect and Pseudo-IMF

Let $x(t)$=2+sin(4πt)+sin(10πt+0.5π) be an inspected signal, where 2 denotes the static component, two sin functions denote the dynamic components. The sampling interval is 0.01s. The sampling time is 2s. Fig.1 shows the decomposition result with EMD. The signal $x(t)$ has been decomposed into five IMFs and a residual. Imf1, imf2, imf3, imf4, imf5 and r5 denote IMFs and residual, respectively. Note that the inspected signal $x(t)$ only contains two frequency components. Therefore, only two among the five IMFs are the desirable IMFs, and the other three IMFs are the undesirable IMFs. In this paper, the undesirable IMF is called pseudo-IMF.

Since pseudo-IMF is not the real frequency component of the inspected signal, it should have weak correlation with the inspected signal. We can use the correlation coefficient of IMF with the inspected signal to judge whether an IMF is the pseudo-IMF. Here, the correlation coefficient can be described as [6]

$$\lambda_i = \frac{N\sum_{n=1}^{N} x(n)c_i(n) - \sum_{n=1}^{N} x(n)\sum_{n=1}^{N} c_i(n)}{\sqrt{\left[N\sum_{n=1}^{N} x^2(n) - (\sum_{n=1}^{N} x(n))^2\right]\left[N\sum_{n=1}^{N} c_i^2(n) - (\sum_{n=1}^{N} c_i(n))^2\right]}} \tag{4}$$

where $x(n)$ denotes the inspected signal, $c_i(n)$ denotes the i-th IMF, N denotes the samples number of the inspected signal, λ_i denotes the correlation coefficient for the i-th IMF with the inspected signal $x(t)$. Set a value, when λ_i less than the value, we can judge the i-th IMF as pseudo-IMF. In fig.1, the correlation coefficients for every IMF with $x(n)$ are 0.6868, 0.5405, 0.0007, 0.0145 and 0.0081, respectively. If λ=0.1, then imf3, imf4 and imf5 can be regarded as pseudo-IMFs.

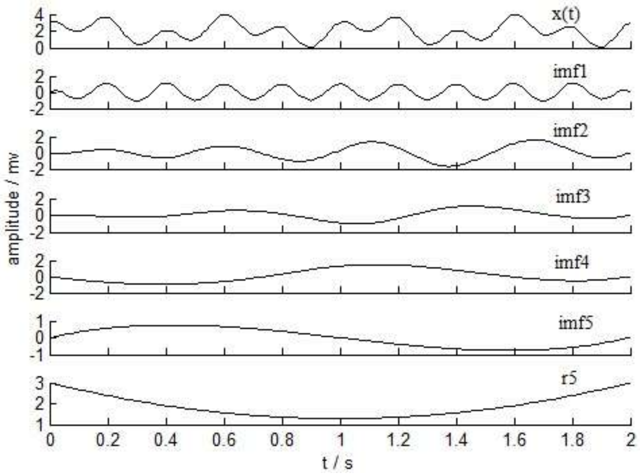

Figure 1. EMD with end points

Inspecting imf1, imf2, imf3, imf4 and imf5 in fig.1, we can find that every IMF is 0 at t=0. At the same time, we study the composition of signal $x(n)$. The dynamic component $\sin(10\pi t+0.5\pi)$ is 1 at t=0. It is clear that imf1 distortion is obvious near t=0.

The procedure of EMD is a kind of interpolating process based on signal extremum points. In practice, it is difficult to know whether two end points of signal are the extremum points. During the decomposition, two end points of signal are often simultaneously regarded as maximum point and minimum point. The envelope lines based extremum points will inevitably deform near the end points of signal, which results in distortion of IMFs derived from envelope lines. With the decomposition of signal, IMFs' distortion will extend from signal end to signal middle. When signal is short, IMFs' distortion is especially obvious. The phenomenon of IMFs' distortion near the end points is called end effect in this paper.

End effect directly influences EMD application. End points extending methods are often employed to control end effect. The basic principle of end points is to predict location and magnitude of data points outside of signal end points. Mirror method, statistical method, polynomial fitting method and neural network method are used to extend end points in [5]. In this paper, PSO is employed to estimate the data points outside of signal end points.

IV. Particle Swarm Optimization

Particle swarm optimization (PSO) is a global optimization method firstly proposed by J.Kennedy and R.C.Eberhart [7] in 1995. PSO searches the optimal solution by simulating the behavior of bird flock looking for food, and has been widely used to solve nonlinear, non-differentiable, multi-modal problems.

PSO initializes a group of random particles, each of which represents a potential solution to a problem. The performance of each article is assessed by the fitness function previously constructed. Each particle adjusts its flying according to its own flying experience and its companions' fly experience in solution space. In every generation particle swarm, each particle tracks two best values. The first one is the optimal solution found by particle own so far. The second one is the optimal solution found by whole particle swarm so far. By iteration research, the optimal solution can be obtained. The basic principle and steps of PSO are described in detail in [8].

For improving the performance of PSO, a weight is introduced in practice. The velocity equation of standard PSO can be modified as [9]

$$v_{id}^{t+1} = \alpha v_{id}^{t} + c_1 r_1 (p_{id}^{t} - x_{id}^{t}) + c_2 r_2 (p_{gd}^{t} - x_{id}^{t}), \quad (5)$$

where α is a weight which affects the proportion of the particle's previous velocity in current velocity. If the valuate of α is large, then the global search performance of PSO is good and the local search performance is bad, and vice versa. A good weight will improve the performance of PSO and reduce the iteration times; v_{id}^t and x_{id}^t are the velocity and position of the i-th particle at t time; c1 and c2 are two positive constants (usually c1= c2=2); r1 and r2 are two random numbers in the range [0, 1]; p_{id}^t and p_{gd}^t are individual optimal position and global optimal position at t time

For a good balance between global optimization and local optimization of PSO, an inertia weight [10] is presented as following

$$\alpha = \alpha_{start} - \frac{\alpha_{start} - \alpha_{end}}{\tau_{max}} \times \tau , \qquad (6)$$

where τ_{max} is the maximum iteration number, τ is the current iteration number, α_{start} and α_{end} are the initial weight and final weight, respectively. At beginning, a large weight is good for global optimizing and fast search the region containing the optimal solution. The weight becomes small and small with iteration and the local searching performance of PSO is gradually improved, the global optimal solution can be obtained at last.

V. Improved EMD

Improved EMD based PSO can be summarized as follows:

1) Analyzing the frequency components of decomposed signal, and constructing the parameter equation of decomposed signal

$$f(t) = w + \sum_{i=1}^{n} A_i \sin(2\pi f_i t + \varphi_i) \qquad (7)$$

where *t* is the sampling time, *w* is the static component, *n* is the number of the dynamic components, A_i and φ_i are the amplitude and initial phase of the $i-th$ dynamic component respectively, f_i is the $i-th$ frequency component obtained by spectral analysis.

2) Constructing the objective equation

$$G(\boldsymbol{X}) = \min \sum_{i=1}^{n} (f(iT) - y(iT))^2 , \qquad (8)$$

where T is the sampling interval, n is the sample number of signal $y(t)$, $\boldsymbol{X} = [w, A_i, \varphi_i \mid i = 1, 2, \cdots, n]^T$ is the parameter vector.

3) Using PSO to get the optimal estimation $\hat{\boldsymbol{X}} = [\hat{w}, \hat{A}_i, \hat{\varphi}_i \mid i = 1, 2, \cdots, n]^T$, and reconstructing signal $y(t)$, and extending the data points outside of end points of the inspected signal;

4) Decomposing the extended signal by EMD, and getting IMFs and residue of extended signal;

5) Subtracting the extended data points from the obtained IMFs and residue, and getting the true IMFs and

residue contained in the inspected signal;

6) Calculating the correlation coefficient between each IMF and inspected signal, and judging the pseudo-IMF according to the correlation coefficient, pseudo-IMFs and residue added together to form the final residue of inspected signal.

VI. Simulation

Let $f(t)=1+0.2\sin(2\pi 2t+\pi/3)+0.15\sin(2\pi 5t+\pi/5)+0.1\sin(2\pi 10t+\pi/6)$ is an inspected signal, where three sine functions denote the dynamic components contained in $f(t)$ and 1 denotes the static component. The sampling interval is 0.001 s and sampling time is 0.2 s. For the dynamic component $0.2\sin(2\pi 2t+\pi/3)$, only 0.4 multi cycles can be acquired in 0.2 sampling time. In t = [0, 0.2], there are only 4 extreme points in signal $f(t)$. Using standard EMD method directly, the decomposition result is shown in fig.2. There exists obvious distortion near end points of imf1 and imf2. Inspecting signal $f(t)$ at the same time, we can see that dynamic component $0.2\sin(2\pi 2t+\pi/3)$ has not been extracted.

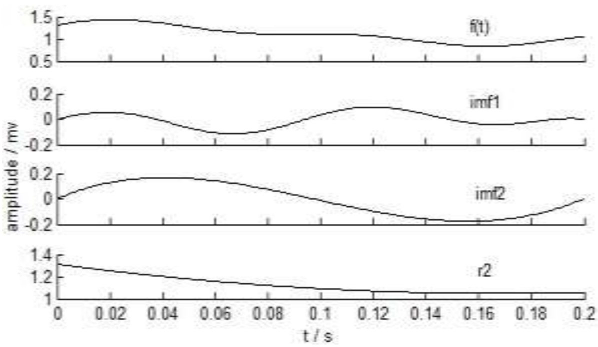

Figure 2 Decomposition result of standard EMD

The frequencies obtained by spectrum analysis are regarded as the frequency components of the dynamic components. According to signal model (7), the amplitudes and initial phases of the dynamic components and the static component can be regarded as the unknown parameters. Combining the sampling signal with model (7), the objective function (8) is constructed. The population size is 100, and maximum iteration is 600, and convergence accuracy is 0.00001. The inertia weight (6) is used, and initial weight is 0.9, and final weight is 0.4. The objective function is employed as the fitness function to assess the particles' performance.

According to the estimations acquired, we can extend the signal to be decomposed by calculating the values of data points outside signal endpoints. Fig.3 shows the extension results at t=[-0.4 0] and t=[0.2 0.6]. It is clear that two extension parties are almost entirely consistent with original signal. We use EMD method to decompose the extended signal, and compute the correlation coefficients according to equation (4). The pseudo-IMFs and residue are added to form new residue. The new residue is regarded as the real residue of original signal. Fig.4 shows the last decomposition result. Compared with decomposition result shown in Fig.2, the distortions of IMFs obviously become small near the endpoints, and extract the lowest frequency component of original signal. EMD performance is improved obviously for short signal decomposition.

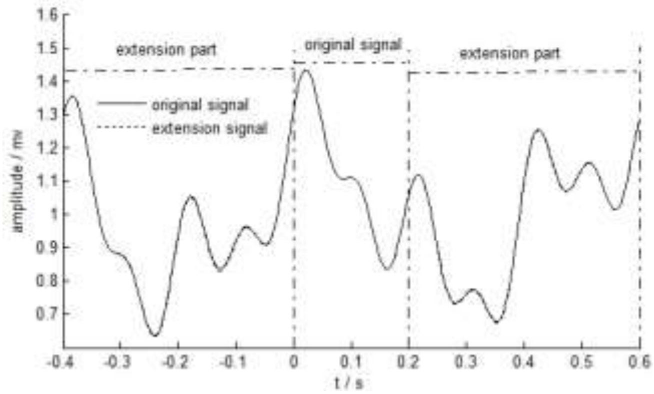

Figure 3 Signal extension result

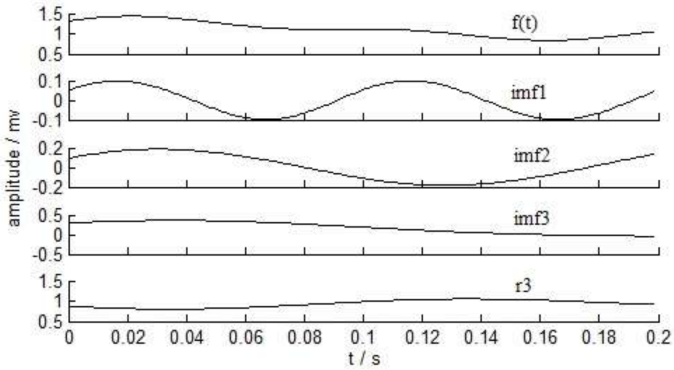

Figure 4 Decomposition results using improved EMD

VII. Conclusion

This paper proposes an improved EMD for decomposing short signal. The procedure can be summarized as four steps: (1) Analyzing the frequency components of inspected signal, and regarding the static component, amplitudes and initial phases as unknowns, and constructing the parameter equation of signal. (2) Using PSO to obtain the optimal estimations of unknown parameters, and extending the inspected signal. (3) Using EMD to decompose the extended signal and get IMFs and residue. (4) Subtracting the extended parties from the obtained IMFs and get the IMFs of original signal, then calculating the correlation coefficients of each IMF and the original signal, the pseudo-IMFs and residue are added to form the residue of original signal. The simulation results illustrate the effectiveness of proposed method.

VIII. Acknowledgment

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